# Variability of silver fir (Abies alba Mill.) cones - variability of cone parameters 

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#### Abstract

This study aimed at determining the shape of closed silver fir cones from the Jawor Forest District (Wroclaw), based purely on measurements of their length and thickness. Using these two parameters, the most accurate estimations were achieved with a fourth-degree polynomial fitting function. We then calculated the cones' surface area and volume in three different ways: 1) Using the fourth-degree polynomial shape estimation, 2) Introducing indicators of compliance ( $\mathrm{k} 1, \mathrm{k} 2, \mathrm{k} 3$ ) to calculate the volume and then comparing it to its actual value as measured in a pitcher filled with water, 3) Comparing the surface area of the cones as calculated with the polynomial function to the value obtained from ratios of indicators of compliance (ratios k4 and k5). We found that the calculated surface area and volume were substantially higher than the corresponding measured values. Test values of cone volume and surface area as calculated by our model were $8 \%$ and $5 \%$ lower, respectively, compared to direct measurements. We also determined the fir cones apparent density to be $0.8 \mathrm{~g} \cdot \mathrm{~cm}-3$ on average. The gathered data on cone surface area, volume and bulk density is a valuable tool for optimizing the thermal peeling process in mill cabinets to acquire high quality seeds.


Keywords: cone, scales, area, volume, shape

## 1. Introduction

Silver fir (Abies alba Mill.) grows in southern and middle Poland, reaching its northern limit of distribution. This species is most numerous in Karpaty Mountains on a height of 5001100 m a.s.l., in Sudetes, Świętokrzyskie Mountains and on Roztocze (Gunia, Kowalski 1968; Wilczkiewicz 1976; Gunia 1986; Sabor et al. 1999; Barzdajn 2009; Bednarek 2002; Sugiero 2005; Szeligowski et al. 2011; Bis, Dobrowolska 2012).

When grown in canopy, the Silver fir bears seeds at the age of 70 years, and in open spaces - at the age of around 30 years (Załęski 1995). This species bears seeds every 3-4 years. Ripe cones of Silver fir are grey-brown, have a length of $10-17 \mathrm{~cm}$ and a thickness 3-5 cm (Tyszkiewicz 1949; Boratyński 1983; Suszka 1983; Schütt 1991; Tracz, Barzdajn 2007; Jaworski, Paluch 2007). Authors Gudeski (1966), Kočiová (1974), Nanu (1977) or Ballianand Čabaravdić (2005) have written about the parameters of cones and seeds of fir from other regions. The fir's cones grow vertically on branches, and after ripening, they fall apart into scales and seeds. The scales and seeds fall down on soil and the axis remains on the tree.

Cones are collected from standing trees by hand before they fully ripen. Obtaining the fir's seeds from cones does not involve using high temperatures and special peeling devices like in the case of common spruce (Picea abies (L.) H. Karst), Scots pine (Pinus sylvestris L.) or European larch (Larix decidua Mill.). According to the instructions of collection and storage of gene resources (Forest Gene Bank in Kostrzyca 2007), the cones should be placed in boxes with perforated bottoms in a ventilated hall with a temperature of $20^{\circ} \mathrm{C}$. Cones, during storage are raked, and they dry and fall apart partially into scales, seeds and axes. Finally, the material is subjected to crushing and then separation in a seed drum sieve. The process of fir's cones peeling can be mechanized, but in order to do so, besides acquaintance of temperature conditions, acquaintance of the cones' structure is needed.

The authors, in a few publications, have described the external parameters of cones and scales, the mass of the seeds or their wings, their mutual dependence (Politi et al. 2011; Jaworski, Paluch 2007; Illoul-Hachi et al. 2015), and also the influence of environment on population or hybrids (Kobliha et al. 2014).In the publications, the parameters of cones of other species were

Submitted: 21.11.2015, reviewed: 10.03.2016, accepted after revision: 22.04.2016.
described in detail, inter alia, common spruce (Kulej, Skrzyszewska 1996), along with showing the dependence between their dimensions (Barzdajn 1996) and the environment from which the material came (Illoul-Hachi et al. 2015). Buraczyk (2009), in studies on cones of common spruce, drew attention to the influence of cones' size and scales' location on the speed of opening of the cones and the seed's release.

The aim of this research was to make an attempt to establish the shape of the cones and to elaborate the most accurate model for the calculation of surface and volume of closed cones of Silver fir. The knowledge of the described parameters can help in the optimization of conditions for peeling realization, while taking into account the biological characteristics of the seeds.

## 2. Material and methods

In this research, we used closed cones of Silver fir that were collected in the economic seed stands in Jawor Forest Inspectorate (Regional Directorate of State Forests in Wrocław) from 751 regions of the origin.

For each of the 30 randomly chosen cones (Fig. 1), we measured length (h), thickness - the largest diameter of cone


Figure 1. View of the investigated silver fir cones (fot. M. Aniszewska)
(d), mass (m) and number of scales (n). The average humidity of cones was evaluated.

A slide calliper was used for the measurement of length and thickness of closed cones and a laboratory scale WPS 600 was used for the measurement of mass. The accuracy of length and thickness measurement amounted 0.1 cm , and the accuracy of mass measurement was 0.1 g .

On the base of length measurement and diameter additionally measured sequentially every 5 mm , calculated was surface area of each cone. Closed cones were treated as having a lathed shape. The generatrix of the external surface was outlined. The distance of location of cross-section from the base of the cone was adopted as a zero point of the system of coordinates (Aniszewska 2001).The coordinates of location of cross-section and the radius designed for each cone were the base for approximation of the equation, defying the generatrix of the cone's external surface.

The shape function $y=f(x)$ was constant and non-negative on the whole length of the cone (h), therefore the surface area $\left(S_{o b l}\right)$ could be calculated with the use of formula (1):
$S_{o b l}=2 \cdot \pi \int_{a}^{b} y d L=2 \cdot \pi \int_{0}^{h} y \sqrt{1+\left(\frac{d y}{d x}\right)^{2}} d x$
where:
$d L$ - differentia of the shape function
By taking into consideration the fact that the area of the base of this entity was small, we assumed that the side surface was equal to the cone's external surface.

Volume of the cone ( $V_{o b}$ ) was designed using the formula (2):
$V_{o b l}=\pi \int_{0}^{h} y^{2} d x$
The external surface and volume of the examined cones was also calculated by using the commonly known formulas for side area and volume of cylinder $\left(S_{w}, V_{w}\right)$ and cone $\left(S_{s}, V_{s}\right)$, where $d$ is the diameter of cone at the thickest point, $h$ is the length of cone and $l$ - is the generatrix of cone (Fig. 2).
a

b


Figure 2. Geometric models mapping the shape of silver fir cones: a - cylinder, b - cone

In order to calculate the examined values more precisely, it was proposed to introduce a coefficient $\alpha$ to the formulas for area surface $S_{s}(3)$ and volume $V_{s}(5)$ of the cone. Coefficient $\alpha$ was equal to ratio of $h_{1}$ and $h$ (Aniszewska 2001). Symbol $h_{1}$ is the distance from the cone's base to the location of cross-section of its maximum diameter (Table 1). The generatrix of cone-1 is a straight line drawn from the apex through point $d$ defying the maximum diameter, and $d_{1}$-the diameter of the base of the cone (Fig. 2b).
$S_{S}=\pi \cdot \frac{d_{1}}{2} \cdot \sqrt{\left(\frac{d_{1}}{2}\right)^{2}+h^{2}}$
where:
$S_{S}$ - surface area calculated from a cone,
$d_{1}, h-$ as given in Figure 2
The geometrical dependence indicates that:
$r_{1}=h \frac{r}{h-h_{1}}=\frac{r}{1-\alpha}$
where:
$r_{1}$ - radius of the cone base,
$r-$ cone's radius.

$$
\begin{equation*}
V_{S}=\frac{1}{3} \pi \cdot\left(\frac{d_{1}}{2}\right)^{2} \cdot h \tag{5}
\end{equation*}
$$

where:
$V_{S}$ - volume of the cone calculated from a cone.
Additionally, we measured the cone's actual volume ( $V_{r}$ ). For measurement, a measuring cylinder filled with water was used. The volume of the supplanted liquid was adopted as the cone's volume. The measurements were made with accuracy to $1000 \mathrm{~mm}^{3}$. The density of cones was calculated as a quotient of mass and actual volume.

For comparison of the calculated volume values, indicators of compliance were introduced according to models with actual volume: $k_{1}=V_{\text {obl }} / V_{r 2}, k_{2}=V_{w} / V_{r 2}, k_{3}=V_{s} / V_{r 2}, k_{4}$ $=S_{\text {obl }} / S_{w,}, k_{5}=S_{\text {obl }} / S_{s}$.

Descriptive statistics (Statistica 2011) were made for external parameters. The mean value, length of half-interval con-
fidence for mean and minimum and maximum for standard deviation were defined. The average surface area and volume were compared with the F- test of variance analysis. Homogeneity of variance (Levene's test) and correspondence with normal distribution was also tested. For testing the normality of distribution of dependent variable, Shapiro-Wilk test was used. All analysis were made on significance level $\alpha=0.05$.

## 3. Research results

### 3.1. Characteristic parameters of cones

The length ( $h$ ) and thickness values (d) of examined cones of Silver fir are given in Table 1. The length of cones ranged from 12.4 to 19.7 , having an average of $16.43( \pm 0.67) \mathrm{cm}$. The thickness ranged from 3.1 to 4.2 , with an average of $3.75( \pm 0.42)$ cm . The number of scales in a cone amounted from 125 to 219, average being $185( \pm 7.60)$. A significant dependence between thickness and length of cones, number of scales $(\mathrm{n})$ and those two characteristics was showed. Equations of linear correlation and coefficients of determination are given below.
$d=0.132 h+1.571$

$$
\begin{equation*}
R^{2}=0.650 \tag{6}
\end{equation*}
$$

Increase of length of cone by 1 cm caused an increase in its volume by 1.3 mm .
$n=9.732 h+25.06 \quad R^{2}=0.523$
Equation (7) indicated that with each centimetre there were 10 more scales.

The mass of closed, fresh cones amounted to an average of $105.91 \mathrm{~g}( \pm 8.85)$ and ranged from 48 to 142 g (Table 1). Average humidity of Silver fir cone's right after harvesting amounted to an average of $110 \%$.

Density for the examined part of cones amounted from 0.68 to $0.90 \mathrm{~g} \cdot \mathrm{~cm}^{-3}$, having an average of $0.81 \mathrm{~g} \cdot \mathrm{~cm}^{-3}( \pm 0.02)$.

### 3.2. Surface and volume of closed cones

After many trials, the fourth-degree polynomial was chosen as the best representative of the shape of cones. The determination coefficient $R^{2}$ obtained ranged from 0.949 to

Table 1. Characteristic parameters of silver fir cones

| No. of cone | Lenght | Distance from <br> the base | Coefficient | Thickness | Number <br> of scales | Weight | Actual <br> volume |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $h$ | $h_{1}$ | $\alpha=h_{l} / h$ | $d$ | $n$ | $m$ | $V_{r z}$ |
|  | cm | cm | - | cm | szt | g | $\mathrm{cm}^{3}$ |
| 1 | 14.4 | 6.0 | 0.42 | 3.5 | 187 | 84.3 | 100 |
| 2 | 17.6 | 7.0 | 0.40 | 3.8 | 184 | 109.0 | 136 |


| No. of cone | Lenght | Distance from the base | Coefficient | Thickness | Number of scales | Weight | Actual volume |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $h$ | $h_{1}$ | $\alpha=h_{l} / h$ | $d$ | $n$ | $m$ | $V_{r z}$ |
|  | cm | cm | - | cm | szt. | g | $\mathrm{cm}^{3}$ |
| 3 | 15.8 | 6.5 | 0.41 | 3.4 | 188 | 81.6 | 100 |
| 4 | 19.7 | 6.5 | 0.33 | 4.0 | 216 | 141.8 | 180 |
| 5 | 17.2 | 5.5 | 0.32 | 3.8 | 199 | 110.9 | 150 |
| 6 | 15.7 | 5.0 | 0.32 | 3.5 | 182 | 97.6 | 115 |
| 7 | 12.5 | 5.0 | 0.40 | 3.5 | 157 | 76.7 | 88 |
| 8 | 17.0 | 3.5 | 0.21 | 4.2 | 202 | 122.7 | 160 |
| 9 | 14.4 | 3.5 | 0.24 | 3.4 | 155 | 65.6 | 80 |
| 10 | 13.7 | 3.5 | 0.26 | 3.20 | 162 | 73.1 | 85 |
| 11 | 16.8 | 4.0 | 0.24 | 4.10 | 190 | 129.3 | 160 |
| 12 | 18.2 | 7.0 | 0.38 | 3.90 | 192 | 110.1 | 145 |
| 13 | 18.5 | 4.0 | 0.22 | 3.90 | 202 | 125.0 | 150 |
| 14 | 17.5 | 4.0 | 0.23 | 4.10 | 197 | 132.1 | 170 |
| 15 | 17.2 | 6.0 | 0.35 | 3.90 | 186 | 100.6 | 148 |
| 16 | 14.1 | 4.0 | 0.28 | 3.65 | 152 | 90.5 | 110 |
| 17 | 16.5 | 5.0 | 0.30 | 3.60 | 196 | 101.3 | 130 |
| 18 | 17.8 | 5.0 | 0.28 | 4.2 | 198 | 136.2 | 160 |
| 19 | 18.0 | 4.0 | 0.22 | 4.2 | 197 | 136.3 | 160 |
| 20 | 17.5 | 4.5 | 0.26 | 3.8 | 206 | 119.7 | 160 |
| 21 | 16.6 | 4.5 | 0.27 | 4.0 | 179 | 135.5 | 150 |
| 22 | 17.0 | 5.0 | 0.29 | 3.8 | 177 | 105.4 | 135 |
| 23 | 17.1 | 7.0 | 0.41 | 4.0 | 192 | 115.2 | 145 |
| 24 | 15.2 | 6.0 | 0.39 | 3.5 | 176 | 92.0 | 110 |
| 25 | 17.0 | 5.0 | 0.29 | 4.0 | 174 | 111.4 | 140 |
| 26 | 12.4 | 4.0 | 0.32 | 3.1 | 125 | 48.2 | 60 |
| 27 | 14.7 | 4.5 | 0.31 | 3.5 | 171 | 80.8 | 90 |
| 28 | 16.8 | 5.0 | 0.30 | 3.7 | 208 | 120.6 | 140 |
| 29 | 17.2 | 3.5 | 0.20 | 3.8 | 179 | 94.0 | 120 |
| 30 | 18.7 | 4.0 | 0.21 | 3.9 | 219 | 129.7 | 182 |
| Mean | 16.43 | 4.93 | 0.30 | 3.75 | 184.93 | 105.91 | 131.97 |
| Standard deviation | 1.79 | 1.12 | 0.07 | 0.29 | 20.36 | 23.70 | 31.67 |
| Min | 12.4 | 7.0 | 0.20 | 3.10 | 125 | 48.2 | 60 |
| Max | 19.7 | 3.5 | 0.42 | 4.20 | 219 | 141.8 | 182 |

$\alpha, h_{1,}, h-$ as in Figure 2
0.996 , with an average of 0.980 . The general formula for the shape of generatrix of cones was as follows:
$y=A x^{4}+B x^{3}+C x^{2}+D x+E$,
gdzie $x \in(0, h)$.
The mean value, standard deviation and minimum and maximum values of coefficients from A to E are given in Table 2. The exemplary course of changes of generatrix for cones is given on Figure 3.

The designed equations of generatrix of individual cones allowed for surface area $S_{o b l}$ and volume calculation $V_{o b l}$ (Table 3).

Due to a large variability of coefficients $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and E of polynomial for individual cones, despite significant dependence on length (equation 9-12) and thickness, there is no practical possibility of using this equation for the calculation of volume and surface area of any cone of Silver fir when only its basic parameters ( $d$ and $h$ ) are known.

$$
\begin{array}{ll}
A=5 \times 10^{-9} h-1 \times 10^{-6} & R^{2}=0.718 \\
B=1 \times 10^{-6} h+3 \times 10^{-4} & R^{2}=0.677 \\
C=7 \times 10^{-5} h-0.026 & R^{2}=0.410 \\
E=0.044 h-1.366 & R^{2}=0.423
\end{array}
$$

For coefficient $D$, no significant dependence on length or thickness of cones was stated.

Values of surface area of cone $\left(S_{o b}\right)$, calculated from formula 1, amounted from 87.30 to $261.30 \mathrm{~cm}^{2}$, with an average of $156.98( \pm 14.11) \mathrm{cm}^{2}$, and values of volume $\left(V_{\text {obb }}\right)$,
according to the formula 2, from 61.22 to $250.23 \mathrm{~cm}^{3}$, with an average of $144.01( \pm 17.13) \mathrm{cm}^{3}$ (Table 3).

The values of surface area of cone $\left(S_{w}\right)$ calculated from computational model of cylinder ranged from 121.05 to $247.56 \mathrm{~cm}^{2}$, with an average of $194.91 \mathrm{~cm}^{2}( \pm 12.51)$ and values of volume $\left(V_{w}\right)$ from 93.82 to $248.96 \mathrm{~cm}^{3}$, having an average $185.05 \mathrm{~cm}^{3}( \pm 16.38)$ (Table 3).

Values of $\alpha$ and $h_{1}$ coefficients for individual cones, used in the calculation of surface area $S_{s}$ and volume $V_{s}$ of cone, are given in Table 1. On an average, the coefficient $\alpha$ amounted to $0.03( \pm 0.03)$ and $h_{1}$ was equal to $4.93( \pm 0.42)$.

Formulas basis which the values of volume and surface area were calculated according to the cone model (given in Table 3), including $\alpha$ value, have been given in equations13 and 14. The given constants have been taken from equations 3-5.

$$
\begin{align*}
& V_{s}=0.534 \cdot d^{2} \cdot h  \tag{13}\\
& S_{s}=1.602 \cdot d \cdot \sqrt{d^{2}+1.96 \cdot h^{2}} \tag{14}
\end{align*}
$$

The surface area of cone $\left(S_{s}\right)$ amounted from 87.78 to $178.58 \mathrm{~cm}^{2}$, having an average of $140.99 \mathrm{~cm}^{2}( \pm 9.01)$ and volume ( $V_{s}$ ) from 63.79 to $169.27 \mathrm{~cm}^{3}$, on an average 125.82 $\mathrm{cm}^{3}( \pm 11.13) \mathrm{cm}^{3}$ (Table 3).

The results of actual volume $\left(V_{r z}\right)$ are given in Table 1. The average actual volume amounted to from 60 to $182 \mathrm{~cm}^{3}$, with an average $131.97 \mathrm{~cm}^{3}( \pm 11.84)$. The dependence of actual volume was calculated on length and thickness of cones (15 and 16, respectively). The increase in length by 1 cm

Table 2. Statistical values of coefficients $A \div E$ form of the equation

| Parameter | $A$ | $B$ | $C$ | $D$ | $E$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Mean | -0.00000038 | 0.0001245 | -0.014904 | 0.752157 | 5.9349 |
| Standard deviation | 0.00000012 | 0.0000254 | 0.001875 | 0.062385 | 1.2206 |
| Minimum | -0.00000070 | 0.0000850 | -0.019570 | 0.602319 | 3.6540 |
| Maximum | -0.00000020 | 0.0001810 | -0.011298 | 0.882000 | 8.6000 |

a

b


Figure 3. Cone silver fir: a - general view, b - cone outline data visualization
caused the increase of actual volume by almost $16 \mathrm{~cm}^{3}$, and in case of thickness, by around $97 \mathrm{~cm}^{3}$.
$V_{r z}=16.2 \cdot h-133.5 \quad R^{2}=0.829$
$V_{r z}=97.5 \cdot d-233.9 \quad R^{2}=0.817$
The values of surface area and volume calculated in three ways were compared with actual values. The average value of $k_{1}$ indicator, defying the relation of calculated volume $V_{o b l}$ to the measured volume $V_{r z}$ amounted to $1.08( \pm 0.07)$. Due to high compliance of the volume calculated $\left(V_{\text {obl }}\right)$ according to the function of fourth-degree polynomial with value measured for cone $\left(V_{r z}\right)$ allowed us to state that the surface $S_{o b l}$ also calculated with the use of this method is a good approximation of the actual value.

Values of examined factors describing relations of examined indicators: $k_{1}, k_{2}, k_{3}, k_{4}$ and $k_{5}$ are given in Table 3.

Value of indicatork ${ }_{2}$, which is a ratio of volume calculated from cylinder model $\left(V_{w}\right)$ to measured volume $\left(V_{r z}\right)$, ranged from 1.23 to 1.63 , on an average $1.41( \pm 0.04)$. It was much higher than $k_{1}$ value, which proves that that there are significant differences between values calculated from cylinder model and measured values.

In order to use the cylinder model for calculation of actual volume $\left(V_{w f}\right)$, the obtained values should be multiplied by 0.709 (equation 17), and for the calculation of surface area $\left(S_{w f}\right)$, the values should be multiplied by 0.810 , which indicates $S_{w}$ and $S_{o b l}\left(k_{4}\right)$ dependence .

$$
\begin{equation*}
V_{w f}=V_{w} / k_{2}=V_{w} / 1.41=V_{w} \cdot 0.709 \tag{17}
\end{equation*}
$$

Table 3. Surface area and volume, and compliance rates for the tested silver fir cones

| No. of cone | Surface <br> area <br> $S_{o b l}$ <br> $\mathrm{~cm}^{2}$ | Volume$V_{o b l}$$\mathrm{~cm}^{3}$ | Surface <br> area$S_{w}$$\mathrm{~cm}^{2}$ | Volume$V_{w}$$\mathrm{~cm}^{3}$ | Surface <br> area$S_{s}$$\mathrm{~cm}^{2}$ | Volume$V_{s}$$\mathrm{~cm}^{3}$ | Compliancerates |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | $k_{1}$ | $k_{2}$ | $k_{3}$ | $k_{4}$ | $k_{5}$ |
|  |  |  |  |  |  |  | - | - | - | - | - |
| 1 | 122.56 | 91.66 | 155.97 | 134.52 | 112.97 | 91.46 | 0.92 | 1.35 | 0.91 | 0.79 | 1.08 |
| 2 | 168.91 | 137.42 | 208.77 | 197.28 | 150.79 | 134.14 | 1.01 | 1.45 | 0.99 | 0.81 | 1.12 |
| 3 | 141.98 | 106.41 | 168.77 | 143.45 | 121.90 | 97.53 | 1.06 | 1.43 | 0.98 | 0.84 | 1.16 |
| 4 | 261.30 | 250.23 | 247.56 | 247.57 | 178.58 | 168.32 | 1.39 | 1.38 | 0.94 | 1.06 | 1.46 |
| 5 | 171.49 | 140.44 | 202.75 | 190.08 | 146.49 | 129.24 | 0.94 | 1.27 | 0.86 | 0.85 | 1.17 |
| 6 | 134.66 | 101.17 | 172.08 | 150.57 | 124.41 | 102.37 | 0.88 | 1.31 | 0.89 | 0.78 | 1.08 |
| 7 | 111.59 | 84.23 | 136.99 | 119.18 | 99.70 | 81.03 | 0.96 | 1.35 | 0.92 | 0.81 | 1.12 |
| 8 | 181.47 | 162.89 | 222.03 | 230.36 | 160.89 | 156.62 | 1.02 | 1.44 | 0.98 | 0.82 | 1.13 |
| 9 | 114.80 | 80.45 | 153.39 | 130.38 | 111.06 | 88.64 | 1.01 | 1.63 | 1.11 | 0.75 | 1.03 |
| 10 | 124.53 | 93.04 | 137.73 | 110.18 | 99.68 | 74.91 | 1.09 | 1.30 | 0.88 | 0.90 | 1.25 |
| 11 | 193.33 | 203.28 | 216.52 | 221.93 | 156.90 | 150.90 | 1.27 | 1.39 | 0.94 | 0.89 | 1.23 |
| 12 | 191.33 | 196.84 | 222.99 | 217.42 | 161.05 | 147.82 | 1.36 | 1.50 | 1.02 | 0.86 | 1.19 |
| 13 | 189.08 | 159.52 | 226.67 | 221.00 | 163.64 | 150.26 | 1.06 | 1.47 | 1.00 | 0.83 | 1.16 |
| 14 | 149.40 | 155.77 | 224.77 | 230.38 | 162.71 | 156.64 | 0.92 | 1.36 | 0.92 | 0.66 | 0.92 |
| 15 | 188.14 | 173.41 | 210.74 | 205.47 | 152.41 | 139.70 | 1.17 | 1.39 | 0.94 | 0.89 | 1.23 |
| 16 | 135.55 | 108.42 | 162.14 | 147.95 | 117.70 | 100.60 | 0.99 | 1.35 | 0.91 | 0.84 | 1.15 |
| 17 | 198.22 | 197.77 | 186.05 | 167.44 | 134.43 | 113.84 | 1.52 | 1.29 | 0.88 | 1.07 | 1.47 |
| 18 | 185.87 | 175.48 | 231.68 | 240.37 | 167.68 | 163.43 | 1.10 | 1.50 | 1.02 | 0.80 | 1.11 |
| 19 | 143.63 | 140.93 | 237.11 | 248.96 | 171.62 | 169.27 | 0.88 | 1.56 | 1.06 | 0.61 | 0.84 |


| No. of cone | Surface <br> area <br> $S_{o b l}$ <br> $\mathrm{~cm}^{2}$ | Volume | Surface <br> area$S_{w}$$\mathrm{~cm}^{2}$ | Volume$V_{w}$$\mathrm{~cm}^{3}$ | Surface <br> area$S_{s}$$\mathrm{~cm}^{2}$ | Volume$V_{s}$$\mathrm{~cm}^{3}$ | Compliancerates |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | $k_{1}$ | $k_{2}$ | $k_{3}$ | $k_{4}$ | $k_{5}$ |
|  |  |  |  |  |  |  | - | - | - | - | - |
| 20 | 159.88 | 131.51 | 209.04 | 198.58 | 151.01 | 135.02 | 0.82 | 1.24 | 0.84 | 0.76 | 1.06 |
| 21 | 202.95 | 223.18 | 208.10 | 208.10 | 150.76 | 141.49 | 1.49 | 1.39 | 0.94 | 0.98 | 1.35 |
| 22 | 180.11 | 170.80 | 202.95 | 192.80 | 146.72 | 131.09 | 1.27 | 1.43 | 0.97 | 0.89 | 1.23 |
| 23 | 159.19 | 161.83 | 215.14 | 215.14 | 155.71 | 146.27 | 1.12 | 1.48 | 1.01 | 0.74 | 1.02 |
| 24 | 150.80 | 135.62 | 164.96 | 142.28 | 119.30 | 96.74 | 1.23 | 1.29 | 0.88 | 0.91 | 1.26 |
| 25 | 178.29 | 167.16 | 210.96 | 208.32 | 152.66 | 141.64 | 1.19 | 1.49 | 1.01 | 0.85 | 1.17 |
| 26 | 93.62 | 61.22 | 121.05 | 93.82 | 87.78 | 63.79 | 1.02 | 1.56 | 1.06 | 0.77 | 1.07 |
| 27 | 117.20 | 79.57 | 159.76 | 137.79 | 115.64 | 93.69 | 0.88 | 1.53 | 1.04 | 0.73 | 1.01 |
| 28 | 152.86 | 131.86 | 195.75 | 181.07 | 141.45 | 123.11 | 0.94 | 1.29 | 0.88 | 0.78 | 1.08 |
| 29 | 87.30 | 124.82 | 205.45 | 195.18 | 148.49 | 132.71 | 1.04 | 1.63 | 1.11 | 0.42 | 0.59 |
| 30 | 119.48 | 173.21 | 229.61 | 223.87 | 165.72 | 152.21 | 0.95 | 1.23 | 0.84 | 0.52 | 0.72 |
| Mean | 156.98 | 144.01 | 194.91 | 185.05 | 140.99 | 125.82 | 1.08 | 1.41 | 0.96 | 0.81 | 1.12 |
| Standard deviation | 37.78 | 45.88 | 33.49 | 43.86 | 24.12 | 29.82 | 0.19 | 0.11 | 0.08 | 0.13 | 0.19 |
| Min | 87.30 | 61.22 | 121.05 | 93.82 | 87.78 | 63.79 | 0.82 | 1.23 | 0.84 | 0.42 | 0.59 |
| Max | 261.30 | 250.23 | 247.56 | 248.96 | 178.58 | 169.27 | 1.52 | 1.63 | 1.11 | 1.07 | 1.47 |

$S_{\text {obl }}, V_{\text {obl }}-$ Surface area and volume of cones according to formula 1
$S_{w}, V_{w}$-Surface area and volume of cones according to cylinder model
$S_{s}, V_{s}-$ Surface area and volume of cones according to cone model

The recalculated actual values for surface area $S_{f v}$ and volume $V_{\text {fw }}$, amounted to an average $157.88( \pm 10.13) \mathrm{cm}^{2}$ and $131.20( \pm 11.61) \mathrm{cm}^{3}$, respectively. The graphical comparison of examined values (surface area and volume) is shown on Figure 4.

By using variance analysis, significant difference was found between $S_{w}$ and remaining surface area, and between $S_{w f}$ and $S_{s}$ ( $p=0.013$ ) and $S_{w}(p<0.05)$. A significant difference for $p<0.05$ was found between $V_{w}$ and all other examined volume calculation models, when all volume values were compared. Significant dependence for these values was also confirmed by the Levene's test for homogeneity of variance performed for the examined calculation models. It was found that values of volume and surface area have normal distribution.

Value of $k_{3}$ indicator amounted to an average of 0.96 $( \pm 0.03)$. The value of volume calculated from cone model (equation 14) was smaller by almost $5 \%$ from the actual volume $V_{v z}$. On the other hand, $k_{5}$ indicator had an average value of $1.12( \pm 0.07)$. Surface area calculated from modified cone model (equation 13) was on average smaller by $8 \%$ from the surface area $S_{o b p}$ recognized as actual.

## 4. Discussion

While comparing the results of external parameters of the examined Silver fir cones, it was noticed that they were within the range given by other authors (Barzdajn 2009). They were most similar to the parameters obtained for the Silver fir cones from Romania by Nanu (1977). The length of cones ranged from 7.0 to 19.5 cm , and the thickness ranged from 2.9 to 4.6 cm . Similar results were also given by Kočiová (1974) who described the cones from Slovakia.

By knowing the length and thickness of a cone, the surface area and volume of closed cones can be calculated. The proposed model of a polynomial of fourth-degree was used only for description of the cone's shape and for calculation of the surface area and volume of the closed cones. An attempt to apply the polynomial to cones of different parts, with the use of average values of $A, B, C, D$ and $E$ coefficients of equation, did not succeed because it gave much inflated results. The proposed second and third way of calculating surface area and volume of a cone with the use of cylinder or cone turned out to be more useful. The values of volume calcula-


Figure 4. Comparison of mean values, standard errors and standard deviations for the test computational models: $a$ - surface area, $b$ - volume: $S_{\text {obp }} V_{\text {obl }}$ - calculated from the formula 1 and $2, S_{w} V_{w}-$ calculated according to the formula on the cylinder, $S_{s}, V_{s}$ - calculated according to the formula on a cone $(13,14), S_{w p} V_{w f}$ - calculated according to the formula for the inclusion of a fixed cylinder
ted with the use of first of mentioned above entities had to be multiplied by a constant 0.709 . After recalculating, the obtained values of volume were compared with the actual values of volume designed with the use of hydrometric method $\left(V_{r z}\right)$. On an average, the ratio of the sizes equalled 1.00 $( \pm 0.03)$, which proves that the model was well matched. In order for the surface area of a closed cone, which was calculated with the use of formula for cylinder $\left(S_{w}\right)$, to be comparable to value of surface area calculated with the use of function of polynomial of fourth-degree $\left(S_{o b l}\right)$, it should be multiplied by a constant 0.810 . As a result, the proportion of calculated areas amounted to an average of $1.04( \pm 0.08)$.

A research conducted for other species, that is Scots pine and common spruce, defined a way of calculating the surface area and volume from the formula for cone (Aniszewska 2001; Gawart, Mikłaszewicz 2000). The values of surface area and the volume of Silver fir cones calculated according to this method, in comparison to surface calculated as a function of polynomial of fourth-degree, and the actual volume were smaller by 8 and $5 \%\left(k_{5}, k_{3}\right)$, respectively. For common spruce, the values of surface area varied by $5 \%$, and values of volume by $10 \%$ (Aniszewska 2001).

The examined parameters of Silver fir cone, such as length, thickness, mass, humidity, surface area, volume and density can be used in programming thermal peeling processes in cabinet kilns in economic conditions for obtaining seeds of good quality.

## 5. Conclusions

1. The shape of cones of Silver fir quite accurately defines the curve, which is a polynomial of fourth-degree. However, due to vary large differences in relation to actual values of obtained coefficients for this polynomial, an average value cannot be obtained and used for the calculation of volume and surface area of any cone, despite significant dependence on length and thickness of a cone (beside $D$ coefficient).
2. The formula for cylinder or cone can be a general calculating model describing surface area and volume of cones. The values of volume calculated using the formula for cylinder should be multiplied by a constant value 0.709 , and in case of surface area, by the constant 0.810 . However, by using the formula for cone, the description introduced should be the coefficient $\alpha=0.3$. Result analysis showed that the surface area and volume differed only by 8 and $5 \%$ respectively, from actual values after introducing $\alpha$ coefficient.

## Conflict of interest

The authors declare lack of potential conflicts.

## Acknowledgements and funding sources

The research was financed from own resources of the Faculty of Production Engineering of Warsaw Agriculture University in Warsaw.

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## Authors' contribution

M.A. - concept , literature review , methodology , measurement, analysis of results, a statistical study, conclusions, writing, proofreading; U.B. - literature review, measurement, proofreading

